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Converging Cylindrical Shocks in Magnetohydrodynamics

G. B. WHITHAM

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CONVERGING CYLINDRICAL SHOCKS IN MAGNETOHYDRODYNAMICS

G. B. Whitham

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Converging Cylindrical Shocks in Magnetohydrodynamics

by

G. B. Whitham

1. Introduction

In this report the method developed by R. F. Chisnell (Ref 1) for treating converging cylindrical shocks in ordinary gas dynamics is applied to the corresponding problem in magnetohydrodynamics. Chisnell's method is approximate and relatively simple; yet it leads to very accurate results. For strong shocks, Chisnell's predictions may be compared with the exact solution first calculated by Guderley (Ref 2). In both cases the law of propagation of the shock is found to be $U \quad r^{-n}$ where U is the shock velocity at a distance r from the axis of symmetry and n is an exponent depending on the ratio of specific heats γ. In Chisnell's theory n is given by

$$n = \frac{\gamma}{\left(1 + \sqrt{\frac{\gamma(\gamma-1)}{2}}\right)\left(2 + \sqrt{\frac{2\gamma}{\gamma-1}}\right)}$$

The values of n in the two theories are shown below for three values of γ (the Guderley values are taken from recent calculations by Butler (Ref 3)):

γ	1.2	1.4	1.666...
n (Chisnell)	0.163108	0.197070	0.228220
n (Guderley)	0.161220	0.197294	0.226054

The accuracy of the approximate method is well confirmed by these results. It should be remarked, however, that the Guderley solution gives also the whole flow behind the shock whereas Chisnell's theory treats only the immediate neighborhood of the shock. On the other hand, Chisnell's method is not limited to strong shocks; it applies over the whole course of the shock towards the centre. This is an important extension since it is found in practice that the final period in which the

strongshock assumptions apply is often quite small. The predictions of the velocity and strength before the shock becomes strong have been checked with the numerical computations of R. B. Payne (Ref 4) and good agreement was found.

In the corresponding treatment of the magnetohydrodynamic case which is given here, a further advantage of the Chisnell method is the possibility of including a general non-uniform state ahead of the shock. In the Guderley type of similarity solution such a non-uniform state must be a power law distribution of density, pressure and magnetic field which is perhaps rather artificial.

The Chisnell method in general deals with the propagation of a shock down a channel whose breadth varies slowly with distance along the channel. The converging cylindrical shock problem is the special case in which the channel breadth is proportional to the distance along the channel. For, any radial line through the axis of symmetry is a particle path and could be replaced by a solid wall; thus, if we consider the part of the shock moving down the narrow sector between any two such radial lines, its motion is identical with that of a shock in a narrow wedge shaped channel with breadth proportional to r.

The propagation of a shock in a non-uniform channel is solved in the following way. Consider first a channel which is of constant breadth B up to a certain point but then changes its breadth by a small amount δB after which it is again uniform. If a shock specified by the pressure ration P travels up to this change in section, a transmitted shock with modified strength will continue down the channel and a weak sound wave will be reflected. By an appropriate matching of the various flow quantities (velocity, pressure, density, etc.) across these waves and the change in breadth, the change in shock strength δP can be expressed in terms of δB in the form

$$\frac{\delta B}{B} = f(P)\delta P \quad (1)$$

Chisnell treats a showly varying channel as a series of successive infinitessimal changes δB and the law of variation of P with B is found, therefore, by integrating (1). This procedure neglects interactions between the reflected waves from successive steps, but an estimate of these shows that their effect is small. The close agreement with the Guderley solution is convincing evidence of this. For the cylindrical shock B is proportional to r and the integral of (1) gives P as a function of r. From the shock relations, the values of the shock velocity, density ratio, etc. can be determined in terms of P.

If we wish to include a variation of density, pressure, and magnetic field in the medium ahead of the shock, jumps in these quantities must be introduced with the jump $\triangle B$ in breadth. These additional changes contribute further terms in (1), but replacing B by r and assuming that the equilibrium distribution of density, etc. is given in terms of r, we still have a first order differential equation for P as a function of r. For the limiting cases of weak and strong shocks this equation is solved explicitly; for intermediate cases it would probably be necessary to compute the solution numerically.

The investigation given here will be based on the Lundquist equations of magnetohydrodynamics (Ref 5) and in particular the shock relations etc. are all taken from the recent report by Friedrichs (Ref 6).

2. Equations of motion and the shock relations

In terms of the velocity vector $\underline{u}$, the magnetic field vector $\underline{H}$, pressure p, density ρ, and entropy S, the Lundquist equations of magnetohydrodynamics are

$$\frac{\partial \underline{H}}{\partial t} + \nabla \times (\underline{H} \times \underline{u}) = 0 \quad , \tag{3}$$

$$\frac{\partial \underline{u}}{\partial t} + (\underline{u} \cdot \nabla)\underline{u} + \frac{1}{\rho} \nabla p + \mu \underline{H} \times (\nabla \times \underline{H}) = 0 \quad , \tag{4}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0 \quad , \tag{5}$$

(6) $$\frac{\partial S}{\partial t} + (\underline{u} \cdot \nabla) S = 0 \quad ,$$

(7) $$p = p(\rho, S) \quad ,$$

where μ is the permeability. The Maxwell equation $\nabla \cdot \underline{H} = 0$ is essentially included in (3), since the divergence of that equation gives $\partial(\nabla \cdot \underline{H})/\partial t = 0$. In this work the polytropic law

(8) $$p = \rho^{\gamma} K(S)$$

is taken for the equation of state (7).

For flows with cylindrical symmetry we assume that $\underline{u}$ is a radial velocity and that all the flow quantities are functions only of time t and distance r from the axis of symmetry. We do not exclude transverse and axial components H_θ and H_z of $\underline{H}$. Indeed they are of primary importance since it is easily shown that the radial component H_r must vanish or the solution becomes trivial. For, if H_r is non-zero, the condition that the θ and z components of $\underline{H} \times (\nabla \times \underline{H})$ must vanish in (4) requires that rH_θ and H_z be independent of r. In turn these lead to artificial forms for u and so the only feasible case turns out to be $H_\theta = H_z = 0$. But then the flow problem is independent of the magnetic field. Thus, we consider here only the case $H_r = 0$.

With the above assumptions of symmetry, equations (3) to (6) may be written

(9) $$\frac{\partial H_\theta}{\partial t} + \frac{\partial}{\partial r}(H_\theta u) = 0 \quad ,$$

(10) $$\frac{\partial H_z}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r H_z u) = 0 \quad ,$$

(11) $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{2}\frac{\mu}{\rho}\frac{\partial}{\partial r}(H_\theta^2 + H_z^2) + \frac{\mu}{\rho}\frac{H_\theta^2}{r} = 0 \quad ,$$

(12) $$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \quad ,$$

(13) $$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial r} = 0 \quad .$$

If $p_o(r)$, $\rho_o(r)$, $\underline{H}_o(r)$, refer to the possible equilibrium states that may exist in the undisturbed medium ahead of the shock, these functions satisfy the relations

$$(14) \qquad p_o + \frac{1}{2}\mu H_o^2 + \mu \int^r \frac{H_{\theta o}^2}{r}\, dr = \text{constant} \; ,$$

$$(15) \qquad S = \text{constant} \quad , \quad p_o \propto \rho_o^\gamma \; .$$

In addition, it will be necessary in this work to have the relations connecting $u(r)$, $p(r)$, $\rho(r)$, $\underline{H}(r)$ in steady isentropic flow. From equations (9) to (13) we may write these in the differential form (which is convenient for later purposes):

$$\frac{dH_\theta}{H_\theta} + \frac{du}{u} = 0 \quad ,$$

$$\frac{dH_z}{H_z} + \frac{du}{u} + \frac{dr}{r} = 0 \quad ,$$

$$u\, du + \frac{dp}{\rho} + \frac{\mu}{\rho}(H_\theta\, dH_\theta + H_z\, dH_z) + \frac{\mu}{\rho} H_\theta^2 \frac{dr}{r} = 0 \; ,$$

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dr}{r} = 0 \quad ,$$

$$dp = a^2 d\rho$$

where a is the sound speed. Solving these equations we have

$$(16) \begin{cases} \dfrac{dH_\theta}{H_\theta} = -\dfrac{du}{u} = \lambda \dfrac{dr}{r} \\[2ex] \dfrac{dp}{a^2\rho} = \dfrac{d\rho}{\rho} = \dfrac{dH_z}{H_z} = (\lambda - 1)\dfrac{dr}{r} \quad , \\[2ex] \text{where} \quad \lambda = \dfrac{a^2 + \mu(H_z^2 - H_\theta^2)\,/\,\rho}{a^2 + \mu(H_z^2 + H_\theta^2)\,/\,\rho} \end{cases}$$

Turning now to the shock relations, the general form given by Friedrichs simplifies in this case since there is no normal

component of the magnetic field. If the subscript 1 denotes values behind the shock and subscript o refers to the equilibrium state ahead of the shock, the appropriate relations are

$$\rho_1(U - u_1) = \rho_o U \quad ,$$

$$\underline{H}_1(U - u_1) = \underline{H}_o U \quad ,$$

$$p_1 + \frac{1}{2}\mu H_1^2 + \rho_1(u_1 - U)u_1 = p_o + \frac{1}{2}\mu H_o^2$$

$$\frac{1}{2}(U - u_1)^2 + \frac{\gamma p_1}{(\gamma-1)\rho_1} + \frac{\mu H_1^2}{\rho_1} = \frac{1}{2}U^2 + \frac{\gamma p_o}{(\gamma-1)\rho_o} + \frac{\mu H_o^2}{\rho_o} \quad .$$

It is convenient to express p_1, ρ_1, $\underline{H}_1$, u_1, U in terms of the state ahead of the shock and a single parameter characterizing the shock strength. We choose $\xi = \rho_1/\rho_o$ as the parameter; then,

$$(17)\quad \begin{cases} \rho_1 = \rho_o\xi \quad , \quad \underline{H}_1 = \underline{H}_o\xi \quad , \quad u_1 = U\,\dfrac{\xi-1}{\xi} \\ U^2 = \dfrac{2\xi}{(\gamma+1)-(\gamma-1)\xi}\left[a_o^2 + \dfrac{1}{2}b_o^2\left\{(2-\gamma)\xi + \gamma\right\}\right] \\ p_1 = p_o + \dfrac{2\rho_o(\xi-1)}{(\gamma+1)-(\gamma-1)\xi}\left[a_o^2 + \dfrac{\gamma-1}{4}b_o^2(\xi-1)^2\right] \quad , \end{cases}$$

where a_o is the sound speed $\sqrt{\gamma p_o/\rho_o}$ and b_o is the Alfven speed $\sqrt{\mu H_o^2/\rho_o}$.

In the limiting cases of strong and weak shocks, the relations (17) take simplified forms. Strong shocks (i.e. large values of p_1/p_o) correspond either to small values of $\xi - (\gamma+1)/(\gamma-1)$ or to large values of b_o^2/a_o^2. The former is the usual case found in gas dynamics but it is an interesting point that in magnetohydrodynamics strong shocks also arise when the magnetic field is very strong. In the latter case, the only simplification is that a_o may be set equal to zero in equations (17). But, when $\xi \rightarrow (\gamma+1)/(\gamma-1)$, the shock relations take the simple form

$$(18)\begin{cases} \rho_1 = \frac{\gamma+1}{\gamma-1}\rho_o \quad , \quad \underline{H}_1 = \frac{\gamma+1}{\gamma-1}\underline{H}_o \\ p_1 \sim \frac{2\rho_o}{\gamma+1}U^2 \quad , \quad u_1 \sim \frac{2}{\gamma+1}U \quad . \end{cases}$$

It should be noted that in this limiting case, the expressions for p_1, u_1, ρ_1 in terms of U are independent of the magnetic field.

At the other extreme of weak shocks, $\xi \rightarrow 1$ and we have

$$(19)\begin{cases} \rho_1 - \rho_o \sim \rho_o(\xi-1) \quad , \quad \underline{H}_1 - \underline{H}_o \sim \underline{H}_o(\xi-1) \quad , \\ u_1 \sim c_o(\xi-1) \quad , \quad p_1 - p_o \sim \rho_o\, a_o^2(\xi-1) \quad , \\ U - c_o \sim (\frac{\gamma+1}{4}c_o + \frac{2-\gamma}{4}\frac{b_o^2}{c_o})\ (\xi-1) \end{cases}$$

where $c_o = \sqrt{a_o^2 + b_o^2}$.

3. Interaction of a shock with a small change in channel breadth

As explained in Section 1, the main step in the Chisnell method is to determine the strength, etc. of the shock transmitted at a small change in channel breadth. In view of the subsequent application we now denote the channel breadth by r and consider an increase δr at a certain section. If the initial equilibrium state is non-uniform, the change δr is accompanied by increments in p_o, ρ_o, $\underline{H}_o$. From (14) these increments are related by

$$(20)\begin{cases} \delta p_o + \mu\underline{H}_o\cdot\delta\underline{H}_o + \mu H_{o\theta}^2\frac{\delta r}{r} = 0 \\ \delta p_o = a_o^2\ \delta\rho_o \quad . \end{cases}$$

If the equilibrium field $\underline{H}_o$ is given, p_o and ρ_o are determined and the increments are known in terms of δr.

Let the incident shock be specified by the density ratio ξ.

Let the values of flow quantities behind it be u_1, p_1, ρ_1, $\underline{H}_1$, $a_1^2 = \gamma p_1/\rho_1$, $b_1^2 = \mu H_1^2/\rho_1$; these quantities are all determined in terms of ξ by the shock relations (17). The initial situation is shown in Fig. 1. After the interaction, suppose the transmitted shock is specified by the density ratio $\xi + \delta\xi$. Our problem is to determine $\delta\xi$ in terms of δr.

The values of all the flow quantities behind the transmitted shock can be expressed in terms of $\xi + \delta\xi$ and the equilibrium state $p_o + \delta p_o$, $\rho_o + \delta\rho_o$, $\underline{H}_o + \delta\underline{H}_o$ from the shock relations (17), and we denote the values by $p_1 + \delta p_1$, $\rho_1 + \delta\rho_1$, $\underline{H}_1 + \delta\underline{H}_1$, etc. There is also a contact discontinuity following the transmitted shock, separating particles which passed through the incident and transmitted shocks, respectively, and a weak wave is reflected back down the channel. In all, there are five different regions as shown in Fig 2; a space-time diagram is also included in the figure. By relating the values of flow quantities in adjacent regions in an appropriate fashion, we shall have just enough conditions to determine $\delta\xi$.

All the quantities in region o are known, those in region 1 (with subscript 1) are known in terms of ξ, those in region 4 are $p_1 + \delta p_1$, etc. which are known in terms of ξ and $\delta\xi$. We must first deduce the values in regions 2 and 3.

From 1 to 2, the weak wave results of (19) apply [*], with a modification in the sign of u since it is now measured in the direction opposite to the direction of propagation. Thus, eliminating ξ in (19), we have

$$\rho_2 - \rho_1 = -\rho_1\left(\frac{u_2 - u_1}{c_1}\right), \quad \underline{H}_2 - \underline{H}_1 = -\underline{H}_1\left(\frac{u_2 - u_1}{c_1}\right) \tag{21}$$
$$p_2 - p_1 = -\rho_1 a_1^2 \left(\frac{u_2 - u_1}{c_1}\right),$$

where subscripts refer to the values in the various regions.

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[*] Relations (19) actually apply to any small disturbance independent of whether it is a compression or an expansion wave.

To pass from region 2 to 3, we use the steady isentropic flow relations (16), so that, in particular

$$u_3 - u_2 = -\lambda_1 u_1 \frac{\delta r}{r}$$

But, u is continuous across the contact discontinuity i.e. $u_3 = u_1 + \delta u_1$. Therefore,

$$u_2 = u_1 + \delta u_1 + \lambda_1 u_1 \frac{\delta r}{r} \quad . \tag{22}$$

Now, with u_2 and u_3 known in terms of ξ and $\delta\xi$ it is a simple matter to find all the other quantities. Region 2 is determined from (21) and region 3 is then found using the relations given in (16) which may be approximated to first order in δr as

$$\left\{ \begin{aligned} &\frac{H_{\theta 3} - H_{\theta 2}}{H_{\theta 1}} = \lambda_1 \frac{\delta r}{r} \\ &\frac{p_3 - p_2}{\rho_1 a_1^2} = \frac{\rho_3 - \rho_2}{\rho_1} = \frac{H_{z3} - H_{z2}}{H_{z1}} = (\lambda_1 - 1) \frac{\delta r}{r} \quad . \end{aligned} \right. \tag{23}$$

The final condition to be satisfied is that $p + \frac{1}{2} \mu H^2$ is continuous across the contact discontinuity *; this determines $\delta\xi$. We have

$$p_1 + \delta p_1 + \frac{1}{2} \mu H_1^2 + \mu \underline{H}_1 \cdot \delta \underline{H}_1 = p_3 + \frac{1}{2} \mu H_3^2 \quad . \tag{24}$$

From (21), (22), (23),

$$p_3 = p_2 + (\lambda_1 - 1) \rho_1 a_1^2 \frac{\delta r}{r} = p_1 - \rho_1 a_1^2 \left(\frac{\delta u_1}{c_1} + \lambda_1 \frac{u_1}{c_1} \frac{\delta r}{r} \right) + (\lambda_1 - 1) \rho_1 a_1^2 \frac{\delta r}{r} \quad ,$$

---

* The only other condition across a contact discontinuity is that u is continuous; this was used in obtaining (22).

$$H_{\theta 3} = H_{\theta 2} + \lambda_1 H_{\theta 1} \frac{\delta r}{r} = H_{\theta 1} - H_{\theta 1} \left(\frac{\delta u_1}{c_1} + \frac{\lambda_1 u_1}{c_1} \frac{\delta r}{r}\right) + \lambda_1 H_{\theta 1} \frac{\delta r}{r} ,$$

$$H_{z3} = H_{z2} + H_{z1} (\lambda_1 - 1) \frac{\delta r}{r}$$

$$= H_{z1} - H_{z1} \left(\frac{\delta u_1}{c_1} + \frac{\lambda_1 u_1}{c_1} \frac{\delta r}{r}\right) + (\lambda_1 - 1) H_{z1} \frac{\delta r}{r} .$$

Therefore, substituting in (24), we have

$$(25) \quad \delta p_1 + \mu \underline{H}_1 \cdot \delta \underline{H}_1 + \rho_1 c_1 \delta u_1 + \left(\frac{\rho_1 u_1 c_1^2}{c_1 + u_1} + \mu H_{\theta 1}^2 \frac{c_1 - u_1}{c_1 + u_1}\right) \frac{\delta r}{r} = 0 .$$

All the quantities in this equation are functions of ξ given by (17). Hence it determines $\delta\xi$ in terms of δr.

4. Converging cylindrical shocks

The laws of propagation of converging cylindrical shocks are obtained by integrating (25) i.e. $\xi(r)$ satisfies the first order differential equation obtained by substituting (17) in the equation

$$(26) \quad r \frac{d}{dr} \left(p_1 + \tfrac{1}{2} \mu H_1^2\right) + \rho_1 c_1 r \frac{du_1}{dr} + \frac{\rho_1 u_1 c_1^2}{u_1 + c_1} + \mu H_{\theta 1}^2 \frac{c_1 - u_1}{c_1 + u_1} = 0 .$$

The general equation for ξ is complicated and no particular purpose is served by writing it down here, since it would have to be integrated numerically. We can, however, get the solution explicitly in the limiting cases of weak and strong shocks.

Weak shocks

For a weak shock, the approximate relations (19) may be used in (26) and we obtain

$$r \frac{d}{dr} \left(\rho_o c_o^2 (\xi - 1)\right) + \rho_o c_o r \frac{d}{dr} \left(c_o (\xi - 1)\right) + \rho_o c_o^2 (\xi - 1) = 0.$$

The solution of this is

$$(27) \qquad r \rho_o c_o^3 (\xi - 1)^2 = \text{constant}.$$

The left hand side of this equation is proportional to the energy flux of the wave at distance r. Thus the law of propagation states that the energy flux remains constant and we see that (27) is the appropriate generalization of geometrical acoustics. From (19), the variations in shock velocity and pressure jump are given by

$$(28)\qquad \begin{cases} \dfrac{p_1 - p_o}{p_o} \propto \rho_o^{-\frac{1}{2}} c_o^{-\frac{3}{2}} r^{-\frac{1}{2}} , \\[2ex] \dfrac{U - c_o}{c_o} \propto (\dfrac{\gamma+1}{4}) + \dfrac{2-\gamma}{4} \dfrac{b_o^2}{c_o^2}) \rho_o^{-\frac{1}{2}} c_o^{-\frac{3}{2}} r^{-\frac{1}{2}} , \end{cases}$$

Strong shocks with $\xi \sim (\gamma+1)/(\gamma-1)$

In the limiting case of a strong shock, relations (18) may be used in (26), and we have

$$r \frac{d}{dr} (\rho_o U^2) + \rho_o U \sqrt{\frac{2\gamma}{\gamma-1}}\, r \frac{dU}{dr} + \rho_o \frac{U^2 \gamma}{1 + \sqrt{\frac{\gamma(\gamma-1)}{2}}} = 0 .$$

The solution of this equation is

$$(29)\qquad U \propto \rho_o^{-\alpha} r^{-\beta}$$

where

$$(30)\qquad \alpha = \frac{1}{2 + \sqrt{\frac{2\gamma}{\gamma-1}}} , \quad \beta = \frac{\gamma}{(1 + \sqrt{\frac{\gamma(\gamma-1)}{2}})\ (2 + \sqrt{\frac{2\gamma}{\gamma-1}})} .$$

The corresponding law for the pressure p_1 is

$$(31)\qquad p_1 \propto \rho_o^{1-2\alpha} r^{-2\beta} .$$

The exponent β has exactly the same value as Chisnell found in the absence of a magnetic field. Thus the only effect of the magnetic field for strong shocks of this type is the

possibility of introducing an equilibrium density distribution $\rho_o(r)$ maintained by the field $\underline{H}_o$, and satisfying the equilibrium relations

$$p_o + \frac{1}{2} \mu H_o^2 + \frac{\mu H_{\theta o}^2}{r} dr = \text{constant} \quad , \quad p_o \propto \rho_o^{\gamma} \quad .$$

The exponent $1 - 2\alpha$ in (31) is always positive (provided tnat $\gamma > 1$). Therefore, if ρ_o increases towards the center p_1 will increase faster than in the case $\underline{H}_o = 0$ and vice versa.

Strong shocks corresponding to large values of b_o^2/a_o^2

If $\xi - (\gamma+1)/(\gamma-1)$ is not small but nevertheless the shock is strong because the magnetic field is large, the shock relations (17) show that

$$(32) \quad p_1 + \frac{1}{2} \mu H_1^2 \propto \rho_o b_o^2 \quad , \quad U \propto b_o \quad , \quad u_1 \propto b_o \quad , \quad c_1 \propto b_o \quad ,$$

where the factors of proportionality depend on ξ alone. In general, the relations are still so complicated that an explicit solution of (26) seems unlikely. However, in the special case in which ρ_o and b_o are powers of r, it may be observed that the terms in (26) are homogeneous in r. Hence there is a solution of the equation in which ξ takes a certain constant value. In this case the dependence of p_1, U, etc. on r is given by (32). The existence of this type of solution was pointed by Cathleen S. Morawetz who first found it in her investigations of the similarity solutions of the full equations.

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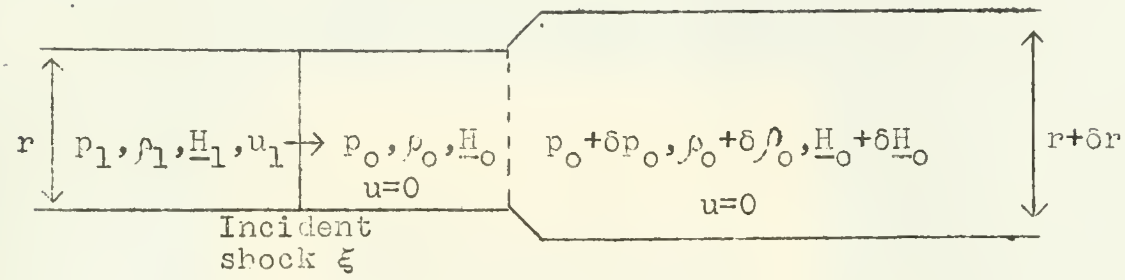
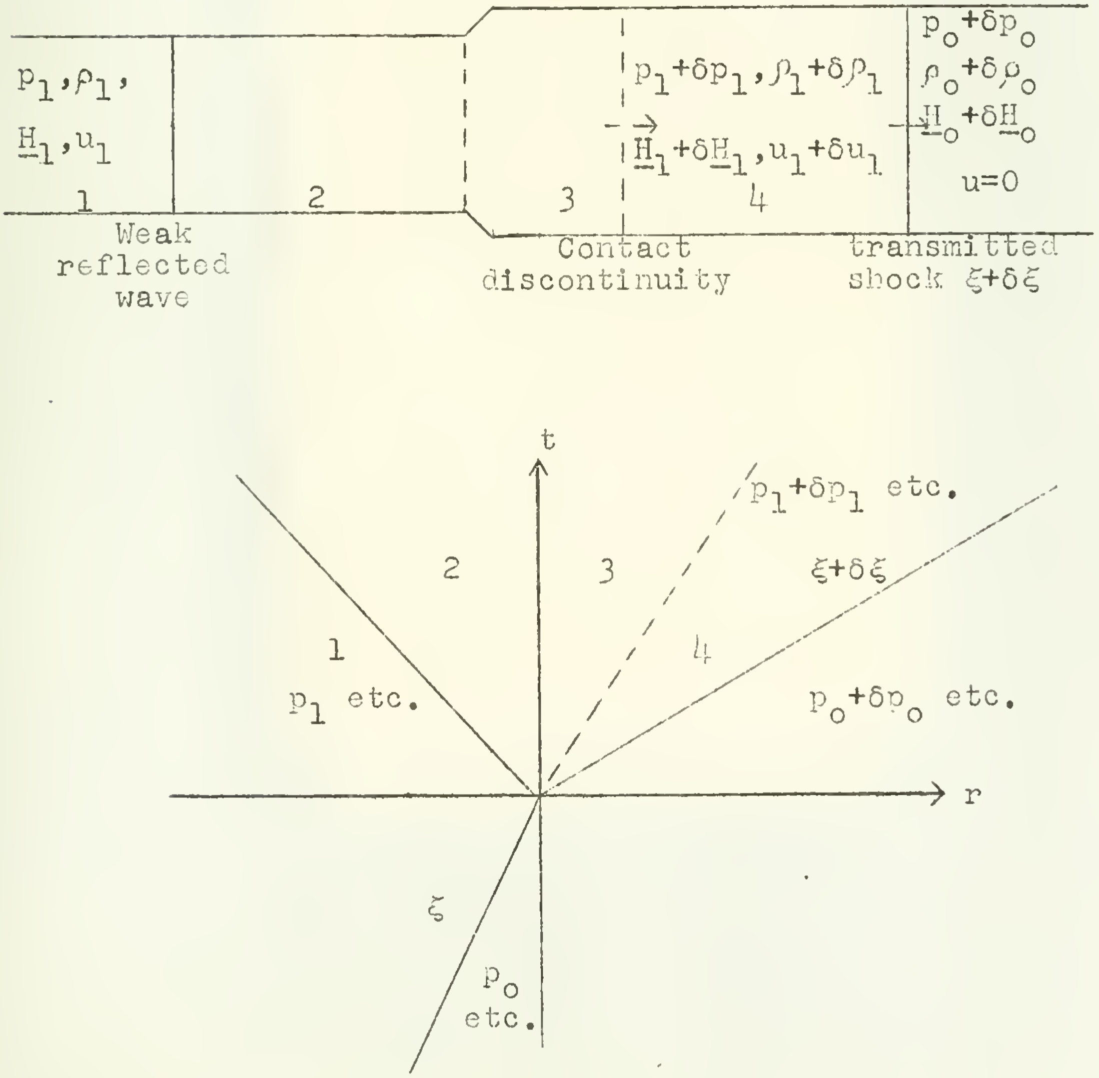

r
$p_1, \rho_1, \underline{H}_1, u_1 \rightarrow$
$p_0, \rho_0, \underline{H}_0$
$u=0$
$p_0+\delta p_0, \rho_0+\delta\rho_0, \underline{H}_0+\delta\underline{H}_0$
$u=0$
$r+\delta r$
Incident shock ξ

Fig. 1


$p_1, \rho_1,$
$\underline{H}_1, u_1$
1
2
3
4
$p_1+\delta p_1, \rho_1+\delta\rho_1$
$\overrightarrow{\underline{H}_1}+\delta\underline{H}_1, u_1+\delta u_1$
$p_0+\delta p_0$
$\rho_0+\delta\rho_0$
$\underline{H}_0+\delta\underline{H}_0$
$u=0$
Weak reflected wave
Contact discontinuity
transmitted shock $\xi+\delta\xi$
t
r
$p_1+\delta p_1$ etc.
$\xi+\delta\xi$
2
3
1
4
p_1 etc.
$p_0+\delta p_0$ etc.
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p_0 etc.

Fig. 2

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